Journal of Basic and Applied Engineering Research
p-ISSN: 2350-0077; e-ISSN: 2350-0255; Volume 2, Issue 23; October-December, 2015, pp. 1989-1993
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Inverse Prediction of Tilt Angle of a Building-Integrated Solar Collector using Golden Section Search Method

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Abstract—*In this paper, the objective is to accurately predict the necessary tilt angle of a Building-Integrated Flat-Plate (BIFP) solar collector for satisfying a given heating requirement. As flat-plate solar collectors are commonly used in building-integrated solar systems, so, the same has been considered in the present study. In order to successfully solve the present problem, an optimization algorithm based on the Golden Section Search Method (GSSM) has been used. The objective function has been represented in the form of a given heat loss factor distribution under different levels of absorber plate temperatures. For a given geometry of the solar collector, subsequently, the necessary tilt angle has been inversely predicted using GSSM and present results have been well-validated with those reported previously. Furthermore, the optimization algorithm has been examined under diverse initial guesses and satisfactory results have been observed in all of the cases. The iterative variation of the estimated parameter and objective function has been analyzed in detail. The present study is proposed to be useful in optimally selecting the necessary tilt angle that will yield a given output and consequently heat loss factor distribution from a BIFP solar collector.*

1. INTRODUCTION

The generation and utilization of energy in a sustainable manner is one of the major requirements in the contemporary scenario. Continuous increase in population directly causes rise in the consumption of energy. At the present rate of fuel consumption, the world is estimated to face a severe energy crunch in the near future. Furthermore, the present energy sources which are being currently used have a harmful impact on the global environment. Eventually, in order to rejuvenate the economy and sustainability in the long term, a new energy resource based on renewable energy is needed. Particularly during winter and in cold regions, a considerable portion of the required energy is utilized for domestic consumption in the form of water and space heating [1]. Due to widespread availability of solar energy, its analysis at present is the most promising and widely-explored resource [2, 3]. Solar energy is cheap, durable, has no impact on global environment and has zero Carbon emission. The future of the solar energy is dependent on the engineering design and the materials of a solar collector.

Among various types of solar collectors, the flat-plate type is the most common type that is widely used in building-integrated solar systems and involves diverse advantages such as simple construction, less maintenance, reasonable efficiency and low production cost [4]. These features also make it suitable for economically weak sections of the society without any compromise in the requirement for which it is desired, as it is well-known that flat-plate collectors are suitable for low to medium temperature heating (approximately upto 100^{o}C) [5].

The study of solar collectors is one of the widely-researched areas under the domain of renewable energy. For evaluating the performance of a solar collector, the estimation of the heat loss factor is one of the significant concerns [6, 7]. For a given application, this is attributed to the fact that a solar collector is required to supply the working fluid at a particular temperature only, and both high and low temperatures are to be avoided, so, for a given range of absorber plate temperature, a particular distribution of heat loss factor is generally required to be satisfied by a collector. The available studies reveal that the heat loss factor from a solar collector is governed by factors such as air gap spacing, collector tilt angle, plate and glass emittance alongwith air properties. Using known values of thermo-geometrical parameters of the collector along with thermal parameters of the ambient, computation of the system behavior is termed as the forward problem [8]. But, a different situation may occur when a few parameters are unknown and a given thermal requirement is to be achieved, then, the problem is referred to as an inverse problem [9]. In order to successfully solve an inverse problem, an optimization algorithm is mandatory for performing the regularization task [10].

It is finally observed that study of inverse problems for solar collectors is very limited and most reported literatures involve with optimizing few parameters [11, 12]. However, it is well-intuitive that the working of a solar collector is dependent upon several aspects such as the plate-glass spacing, size of glass covers, tilt angle alongwith other thermo-geometrical parameters. Among these, the optimization of tilt angle has received significant attention [13-15]. Therefore, in the present work, the tilt angle has been optimized for satisfying a given distribution of heat loss factor in a Building-Integrated flat-Plate (BIFP) solar collector. Due to efficacy in handling single variable and unimodal objective function, a straight line search technique based on the Golden Section Search Method (GSSM) has been used in this work. The following Section describes the formulation and solution methodology of the present problem.

2. FORMULATION AND SOLUTION METHODOLOGY

A single-glazed BIFP solar collector with details presented in Fig. 1 has been considered for the present study. Under steady-state and after considering the sky temperature to be equal to the environment temperature, the heat transfer per unit area can be represented as below [6, 7],

$$\ddot{Q}=\left(h_{r,\,p1}+h_{\text{conv.},\,p1}\right)\left(T_p-T_1\right)=\left(h_{r,1a}+h_{\text{conv.},1a}\right)\left(T_1-T_a\right) \quad (1)$$

where, the second term represents the flux from the absorber plate to the glass cover and the third term pertains to the flux from glass cover to the ambient air. The subscripts 1, p and a are used to indicate the glass cover, absorber plate and environment conditions, respectively. Furthermore, the subscripts r and conv. pertain to radiation and convection, respectively. In Eq. (1), h denotes the heat transfer coefficient in W/(m^2.K) and T indicates the temperature in K.

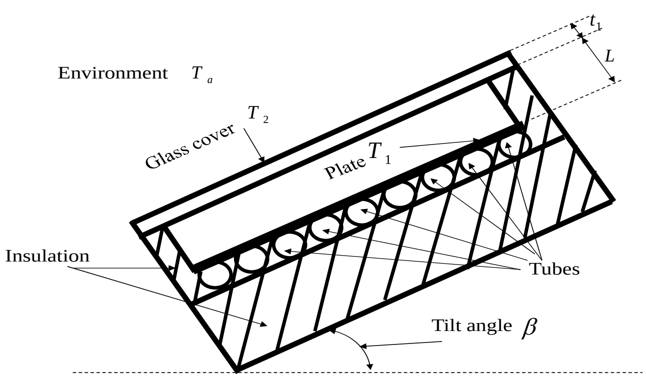

Fig. 1: Schematic of BIFP solar collector

Using known values of various thermo-geometrical parameters and environment properties, the set of governing equations can be iteratively solved to produce the intermediate temperature of the glass cover (T_1). Finally, the total heat loss factor $\left(U_t\right)$is calculated. From the view of accuracy and simplicity, the following correlation has been used [6, 7],

$$U_t^{-1}=\left[\frac{12.75\left\{\left(T_p-T_1\right)\cos\beta\right\}^{0.264}}{\left(T_p+T_1\right)^{0.46}L^{0.21}}+\frac{\sigma\left(T_p^2+T_1^2\right)\left(T_p+T_1\right)}{1/\varepsilon_p+1/\varepsilon_1-1}\right]^{-1}+\left[h_w+\frac{\sigma\varepsilon_1\left(T_1^4-T_a^4\right)}{\left(T_1-T_a\right)}\right]^{-1}+\frac{t_1}{k_1} \quad (2)$$

where, the subscript w denotes the wind condition. In above, k_1 and t_1 represents the thermal conductivity of the glass cover in W/(m.K) and its thickness in m, respectively. Stefan-Boltzmann constant has been represented by $\sigma=5.67\times10^{-8}\ \mathrm{W}/\left(\mathrm{m}^2\cdot\mathrm{K}^4\right)$. The glass cover temperature $\left(T_1\right)$ is expressed by using the following expression [6, 7],

$$T_1=T_a+h_w^{-0.38}\left[0.567\varepsilon_p-0.403+T_p/429\right]\left(T_p-T_a\right) \quad (3)$$

Next, if it is assumed that a given heating requirement is needed from a BIFP solar collector, then for a given plate temperature, both high and low temperatures are needed to be avoided. This requirement is again based upon the fulfillment of a particular distribution of the heat loss factor (say, $\tilde{U}_t$), where all thermo-geometrical parameters are known. However, the collector tilt angle, β is assumed to be unknown. In order to estimate the optimized value of the unknown collector tilt angle, β, the below-mentioned objective function is minimized [3],

$$F=\left(U_t-\tilde{U}_t\right)^2 \quad (4)$$

where, U_t is calculated using guessed values of the unknown (β) which is updated in an iterative process. Consequently, the objective function, F also updates in course of the iterative procedure. For minimizing the objective function, F, GSSM is used, which is briefly discussed below.

GSSM is a direct line search optimization algorithm to estimate either the maximum or the minimum of an objective function by continuously narrowing down the searching range. GSSM is an improvement of the Fibonacci search algorithm, both of which were suggested by Kiefer [16]. GSSM originates from the concept of inverse golden number, r, expressed in the following manner,

$$r=\left(\sqrt{5}-1\right)/2=0.618 \quad (5)$$

The inverse golden number, r always fulfils the following condition,

$$r=\frac{1}{1+r} \quad (6)$$

In GSSM, using two initial guesses/vertices $\left(A \text{ and } B\right)$ with objective function values as $\left(F_A \text{ and } F_B\right)$, two extra points/vertices $\left(V_P \text{ and } V_Q\right)$ with objective functions $\left(V_P \text{ and } V_Q\right)$ to be examined are generated at a distance, r

between the upper and lower ranges $(A \text{ and } B)$ using following expressions [11],

$$V_P = V_A + (1-r)(V_B - V_A) \\ V_Q = V_B - (1-r)(V_B - V_A) \tag{7}$$

If, $F_P > F_Q$, then the section to the right of vertex, V_P (i.e., $V_Q - V_B$) is truncated. Therefore, the new limit becomes $V_P - V_A$ and the point/vertex $V_{P,i}$ turns into $V_{B,i+1}$ for the next iteration, where subscripts i and $i+1$ respectively denote new and previous iteration level. Similarly, if $F_P < F_Q$ then the region to the left of V_Q (i.e, $V_A - V_P$) shall be disregarded and in that case the new interval will be $V_B - V_Q$ and for the next iteration level (i.e., $i+1$) the point $V_{Q,i}$ will be exchanged by $V_{A,i+1}$. In the present study, the above-mentioned iterative process is repeated until the algorithm exceeds 30 iterations, which have been found to be sufficient to lower the value of objective function close to zero. Results and discussion are presented in the next Section.

3. RESULTS AND DISCUSSION

This Section deals with the results and discussion regarding estimation of the unknown parameter (i.e., tilt angle, β of a BIFP solar collector) using GSSM by inverse method. In Fig. 2 validation of the forward method is presented with the relevant results available in the literature [6]. For collector tilt angle, $\beta = 45^{\circ} = 0.7854$ rad., the result corresponds to plate emissivity $\varepsilon_p = 0.1$, air gap spacing, L = 0.025m, glass thickness, 0.003m, glass emissivity, ε_1 = 0.85, thermal conductivity of glass, k_1 = 1.02 W/(m·K), ambient temperature, T_a = 30°C and wind heat transfer coefficient, h_w= 20W/(m^2·K). Satisfactory agreement with the literature result is revealed from Fig. 2.

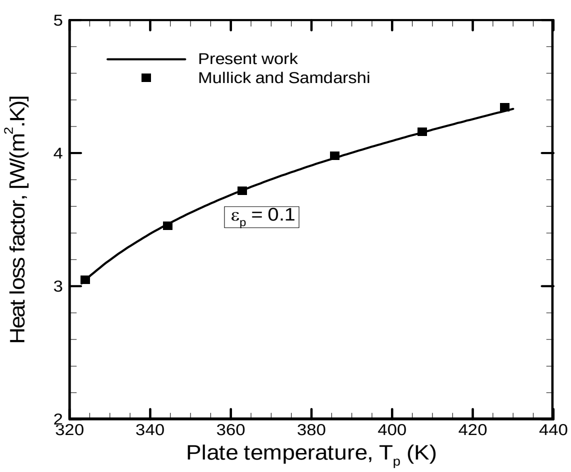

Fig. 2: Validation of the forward method

Next, in inverse problem, the collector tilt angle, β is assumed to be unknown and the task is to determine its value for satisfying a given distribution of the heat loss factor, $\tilde{U}_t$ that was previously computed by solving the forward problem as indicated in Fig. 2. It is worth to mention here that using the present analysis any tilt angle can be obtained satisfying any random distribution of the heat loss factor, but, for the purpose of demonstrating the methodology, results have been presented here for BIFP collector tilt angle, $\beta = 45^{\circ} = 0.7854$ rad.

Table 1: Comparison of exact and estimated values of GSSM-optimized tilt angle for different initial guess.

Initial guess	Exact value	Estimated value
$A_1 = 0, B_1 = \pi/2$	0.7854 rad.	A = B= 0.7854 rad.
$A_1 = \pi/6, B_1 = \pi/2$		A = B= 0.7854 rad.
$A_1 = \pi/6, B_1 = \pi/3$		A = B= 0.7854 rad.

Table 1 presents the optimized values of the unknown collector tilt angle, β for different cases of GSSM involving three different initial guesses. It is observed from Table 1 that the estimated parameter (β) for all 3 cases remains unique with respect to that considered in the forward method (i.e., the exact value).

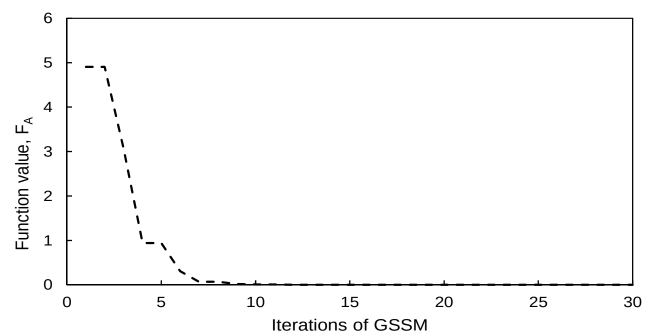

Fig. 3: Variation of objective function for vertex *A* with iterations of GSSM.

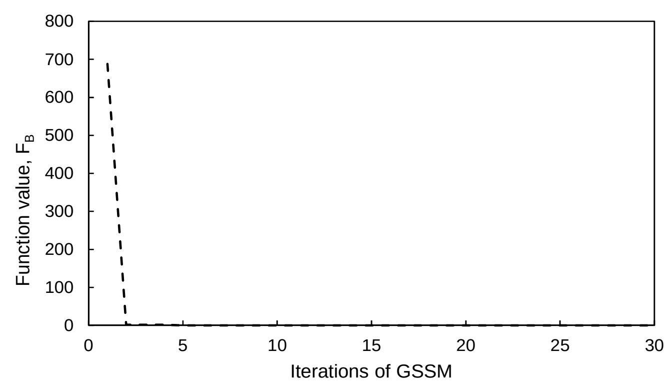

Fig. 4: Variation of objective function for vertex *B* with iterations of GSSM

For case-1 of Table 1, the variation of the objective function, F and estimated parameter (β) with number of iterations of GSSM are presented in Figs. 3-6. It was described previously that in GSSM, the initial guess corresponds to two discrete points (A and B) and subsequently, these two points come closer and eventually merge at the optimized value. So results in Figs. 3-6 have been shown separately for both these points (A and B). It is observed from Figs. 3 and 4 that the objective function, F gradually reduces with number of iterations, and attains a value $F_A=F_B\approx 0$ in about 10 iterations, which does not further considerably vary up to

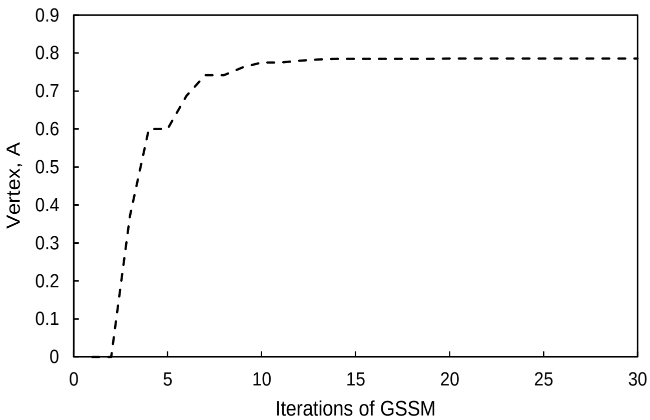

Fig. 5: Variation of guess vertex *A* with iterations of GSSM.

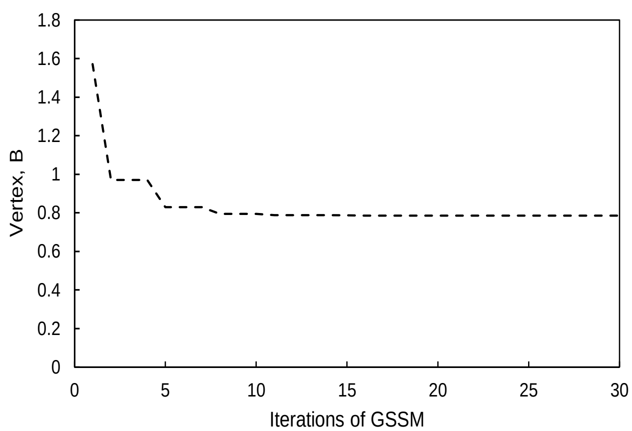

Fig. 6: Variation of guess vertex *B* with iterations of GSSM.

30 iterations. In other words, these two objective functions gradually come close to each other and eventually meet at a single point. A similar observation is also revealed from Figs. 5 and 6 where the two different vertices (A and B) eventually meet at a single vertex that represents the exact and optimized value of the BIFP collector tilt angle, $\beta = 45^{\circ} = 0.7854$ rad. This is again found sufficient for yielding satisfactory reconstruction of the heat loss factor in a BIFP solar collector. The conclusion based upon the present study is described next.

4. CONCLUSION

A Building-Integrated Flat-Plate (BIFP) solar collector has been considered for the present analysis. The optimum tilt angle satisfying a prescribed distribution of the heat loss factor at diverse absorber plate temperatures is estimated using the Golden Section Search Method (GSSM). For demonstration purpose, the reference heat loss factor distribution has been computed by solving a forward problem using some known value of the collector tilt angle and along with other known values of thermo-geometrical parameters. The forward model was well-validated with the reported literature. A very good estimation of the BIFP collector tilt angle was achieved using GSSM under different levels of initial guesses. It is finally concluded that the present GSSM algorithm can be satisfactorily used to predict the appropriate tilt angle of a BIFP solar collector for satisfying any given heating requirement. The present work serves as a basic guideline that may be extended in the future by other researchers by considering multiple objective functions and investigating different optimization algorithms.

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